Definitions of Life Table Functions

The following are definitions of the standard actuarial life table functions. The life table represents a hypothetical cohort of 100,000 persons born at the same instant who experience the rate of mortality represented by q_x, the probability that a person age x will die within one year, for each age x throughout their lives. We assume a uniform distribution of deaths for ages greater than 0. For age 0, we use a separation factor, f_0, which represents the average number of years not lived by those age 0 who die at age 0.

Symbol	Definition
q_x	The probability that a person exact age x will die within one year.
l_x	The number of persons surviving to exact age x.
d_x	The number of deaths between exact ages x and x+1.
L_x	The number of person-years lived between exact ages x and x+1.
T_x	The number of person-years lived after exact age x.
$\mathring{e}_x$	The average number of years of life remaining at exact age x.

The life table functions l_x, d_x, L_x, T_x, and $\mathring{e}_x$ are calculated as follows:

Equation	Range of x (if applicable)
$l_0 = 100{,}000$	
$l_x = l_{x-1} \cdot (1 - q_{x-1})$	x = 1, 2, 3, …
$d_x = l_x \cdot q_x$	x = 0, 1, 2, 3, …
$L_0 = l_0 - f_0 \cdot d_0$	
$L_x = l_x - .5 \cdot d_x$	x = 1, 2, 3, …
$T_x = L_x + L_{x+1} + L_{x+2} + …$	x = 0, 1, 2, 3, …
$\mathring{e}_x = T_x / l_x$	x = 0, 1, 2, 3, …

The following are definitions of additional symbols that are either in the tables or are used to calculate values in the tables:

Symbol	Definition
i	The assumed annual rate of interest.
v	The present value of a payment of \$1 due in 1 year. (The present value of a payment of \$1 due in n years is v^n.)
A_x	The present value at age x of a payment of \$1 payable at the end of the year of death.
$\ddot{a}_x$	The present value at age x of a life annuity of \$1 per year, due at the beginning of each year.
$12\ddot{a}_x^{(12)}$	The present value at age x of a life annuity of \$1 per month, due at the beginning of each month.

The following is a list of calculated functions, including commutation functions:

Equation	Notes
$v = 1 / (1 + i)$	Not in life table; just used for calculations.
$D_x = v^x \cdot l_x$	
$C_x = v^{x+1} \cdot d_x$	Not in life table; just used for calculations.
$M_x = C_x + C_{x+1} + C_{x+2} + \ldots$	
$A_x = M_x / D_x$	
$N_x = D_x + D_{x+1} + D_{x+2} + \ldots$	
$\ddot{a}_x = N_x / D_x$	
$12\ddot{a}_x^{(12)} \approx 12 \cdot (\ddot{a}_x - 11/24)$	11/24 is a standard approximation used when converting an annuity due from an annual to a monthly payment frequency.